Quantum Information-based Hyperbolic Neural Network

PhD Position - Mathematics, Image and Applications (MIA) Lab. La Rochelle Université - France M. Berthier – 2024

Scientific Context

The main objective of this thesis project is to build new neural network architectures for Artificial Intelligence, by exploiting the link between quantum information and hyperbolic geometry, with the aim of optimizing computations, and therefore energy costs, but also to interpret neural network operations in terms of information measurement. The motivations for such a study lie in particular in the fact that tree and hierarchical data are ubiquitous in modern Machine Learning applications. Recent works have successfully demonstrated the advantages and the soundness of considering such data as immersed in hyperbolic spaces, in particular to greatly reduce the number of parameters required for learning.

The proposed topic is therefore part of the effort to understand the mathematical mechanisms underlying such approaches, so as to be able to theoretically account for the performance of these new hyperbolic learning networks, considering them - this is the strong originality of the project from the point of view of quantum information.

The review article *"Hyperbolic Deep Neural Networks: A survey"*, IEEE PAMI, VOL. 44, No. 12, December 2022, clearly shows the application potential of the new structures discussed in this project. In particular, Table 1, page 10031, reports significant contributions in various contexts from GOOGLE, FACEBOOK, DEEPMIND, HUAWEI, NVIDIA.... The article *"Hyperbolic Deep Reinforcement Learning"* produced by TWITTER researchers in October 2022 also illustrates the application potential for decision-making, game theory and robotics.

From a mathematical point of view, the very rapid development of hyperbolic neural networks has been made possible by the fact, highlighted in 2018 by O. Ganea (in his thesis at ETH Zurich), that the basic operations of a network can be efficiently encoded using algebraic structures known as gyro-structures. These structures, discovered by the theoretic physicists of special relativity, had been somewhat forgotten, and it's quite remarkable to see them reappear once again in the context of Machine Learning. Current works make use of the Möbius gyro-structure to model the Poincaré hyperbolic geometry geometry of data.

It turns out that there is a second gyro-structure, an alternative to the Möbius gyro-structure, the Einstein gyro-structure, which we have shown in our work on color perception that it is intimately linked to the measurements of quantum information known as Lüders transformations, see for example: "Quantum measurement and colour perception measurement and colour perception: theory and application", M. Berthier and E. Provenzi, Proceedings of the Royal Society Society A, Vol. 478, Issue 2258, 2022, or "A quantum information-based refoundation of color perception concepts", M. Berthier, N. Prencipe, and E. Provenzi, SIAM Journal on Imaging Sciences, 15(4), 2022. The geometry induced by Einstein's gyro-structure is Klein's hyperbolic geometry. This close relationship between quantum information and hyperbolic geometry via Einstein's gyro-structure is the keystone of the architectures we're to develop. At present, there are no contributions in Machine Learning or

Depp Learning mentioning or exploiting this relationship.

Research

The PhD student will mathematically configure hyperbolic neural networks based on the correspondence between quantum measurements on the state space of a spin factor and operations of Einstein's gyro-structure. He will also implement such networks in PYTHON for simple cases, such as for MLPs or CNNs.

Funding Duration: 36 months (3 years).

Requirements

Applicants must

hold a Master degree in a relevant discipline: applied mathematics and/or machine learning...
or be expected to hold their Master diploma for the registration step in September/October at the very latest;

- have good programming skills in python;

 have strong theoretical and applied background in mathematics (algebra and linear algebra, optimization, probabilities, Riemannian geometry...) and in machine learning (automatic differentiation, backpropagation, deep learning...);

- have proficiency in written and spoken English language.

Application and contact

Applicants must submit

- a one-page cover letter;
- a curriculum vitae.

Applicants must be prepared to provide two reference letters upon request.

Once we receive your application and it fits well for the position, you will be contacted within two weeks.

Applications should be sent, *in a single PDF document*, with the email subject [PhD application] to: michel.berthier@univ-Ir.fr

Contact: Michel Berthier, Laboratoire MIA, La Rochelle Université, 23, Avenue A. Einstein, BP 33060, 17031 La Rochelle cedex, France – michel.berthier@univ-lr.fr –

Application Deadline: 31 March 2024.